Gapless Color Superconductivity

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We present the dispersion relations for quasiparticle excitations about the color-flavor locked ground state of QCD at high baryon density. In the presence of condensates which pair light and strange quarks there need not be an energy gap in the quasiparticle spectrum. This raises the possibility of gapless color superconductivity, with a Meissner effect but no minimum excitation energy. Analysis within a toy model suggests that gapless color superconductivity may occur only as a metastable phase.

Strongly interacting matter at sufficiently high baryon density and low temperature is in a color superconducting state characterized by a condensate of quark Cooper pairs [1–4]. Such a condensate gives mass to gauge bosons via the Anderson-Higgs mechanism. In addition to the Meissner effect, a superconducting phase is typically characterized by an energy gap 2Δ in the density of quasiparticle states. The gap corresponds to the minimum energy necessary to excite one quasiparticle pair relative to the ground state energy. In a typical superconductor, the Meissner effect is therefore accompanied by the characteristic thermodynamic consequences of a gap, like a specific heat $C_V \sim e^{-\Delta/T}$. In this letter we argue that in a color superconductor which includes pairing between quarks with differing mass, an energy gap is not mandatory despite the presence of a condensate. Thus, one may have *gapless superconductivity* in QCD.

At sufficiently high density, quark matter is in the *color-flavor locked* state (CFL) which involves pairing of the light up and down quarks and the heavier strange quark (u, d and s) [5–8]. In this introduction, we give a model independent argument that for pairing of light with strange quarks the energy gap in the quasiparticle spectrum vanishes if the condensate $\langle us \rangle$ (or $\langle ds \rangle$) is less than of order $M_s^2/4\mu$. Here, μ is the quark number chemical potential, M_s is the μ-dependent effective, or constituent, strange quark mass and $\langle us \rangle$ denotes the proper self energy. This raises the possibility of gapless color superconductivity if the condensate $\langle us \rangle$ is nonzero and sufficiently small in high density QCD.

We begin with the dispersion relations for two noninteracting fermions, one massless (u) and the other (s) with mass M_s [9]. At nonzero μ, the u dispersion relation is $\omega_u = \pm(|\mathbf{q}| \pm \mu)$ and the s dispersion relation is $\omega_s = \pm(\sqrt{|\mathbf{q}|^2 + M_s^2} \pm \mu)$, where we are measuring energy relative to the Fermi energy. In Fig. 1 we plot the positive branches, corresponding to empty states which can be filled by single particle excitations. The two different Fermi momenta are apparent, as are the dispersion relations for u and s quarks at momenta greater than their respective Fermi momenta and for u and s holes at momenta less than their respective Fermi momenta. The beginning of the u anti-particle branch is also visible.
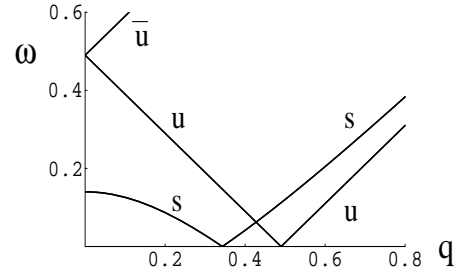

FIG. 1. Dispersion relations for free massless u quarks and s quarks with mass $M_s = 350$ MeV at $\mu = 490$ MeV.

The s and u dispersion relations cross at $\omega_u = \omega_s = M_s^2/4\mu$. We expect this degeneracy to be lifted in the presence of interactions between u and s quarks. Just as a $\langle uu \rangle$ condensate would open a gap $2\langle uu \rangle$ at the u Fermi surface, we expect a $\langle us \rangle$ condensate to open a "gap" as shown in Fig. 2.
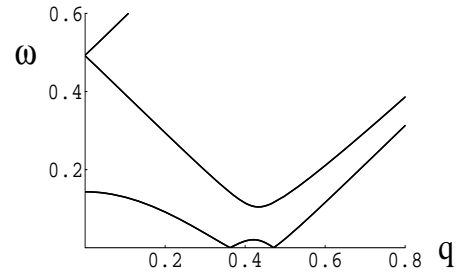

FIG. 2. Dispersion relations for massless u quarks and strange quarks with $M_s = 350$ MeV at $\mu = 490$ MeV in the presence of a $\langle us \rangle = 45$ MeV condensate.

The "u-hole branch" and "s-particle branch" are separated by a "gap" of $2\langle us \rangle$, but this "gap" is *not* at the Fermi energy ($\omega = 0$). This figure depicts a gapless superconductor: $\langle us \rangle \neq 0$, but there are clearly quasiparticle excitations with $\omega = 0$. In Fig. 2, $\langle us \rangle < M_s^2/4\mu$. For larger values of the condensate, the picture must change. We will see that for $\langle us \rangle \gtrsim M_s^2/4\mu$, there is in fact a true gap, as shown in Fig. 3.
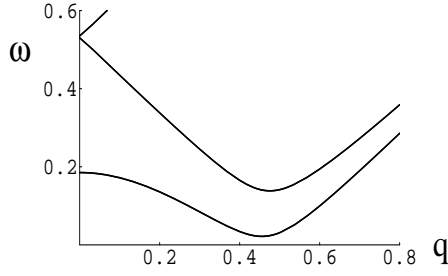
FIG. 3. As in Fig. 2, with $\mu = 525\,\mathrm{MeV}$ and $\langle us \rangle = 85\,\mathrm{MeV}$.

For still larger values of the condensate, we find the dispersion relations of Fig. 4. Note that in these cases, although the superconductor is not gapless, the two dispersion relations have quite different minima, with one gap smaller and the other one larger than the value of the condensate itself.
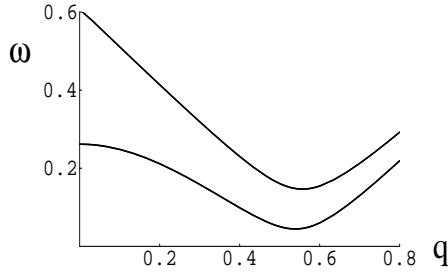

FIG. 4. As in Fig. 2, with $\mu = 600\,\mathrm{MeV}$ and $\langle us \rangle = 100\,\mathrm{MeV}$.

The qualitative lessons of Figs. 1-4 are generic but the figures themselves were derived in a particular model. In Section II, we give a quantitative explanation of the relationship between condensates and the dispersion relations and gaps illustrated in the Figures. In Section III, we present the model within which the specific parameter values used in the Figures are derived. The remaining question, then, is whether a phase of QCD exists in which $\langle us \rangle$ is nonzero but sufficiently small that gapless superconductivity arises as in Fig. 2. We have argued in [7] that if $\langle us \rangle$ is less than the mismatch in Fermi momenta $p_F^u - p_F^s \sim M_s^2/2\mu$ then the $\langle us \rangle$ condensate vanishes at a first order unlocking transition. Because $M_s^2/4\mu < M_s^2/2\mu$, these qualitative arguments suggest that a gapless superconducting phase is never the thermodynamic ground state. This is consistent with what we find in Section III: in our model, gapless superconductivity only arises as a metastable state. However, neither the model nor the qualitative arguments should be trusted to within a factor of two, and it remains to be seen whether QCD admits a stable gapless superconductor CFL phase near the unlocking transition. We close in Section IV with implications for the physics of the CFL phase, and lessons which can be learned from analogue systems in which gapless superconductivity may arise.

II. FROM CONDENSATES TO DISPERSION RELATIONS AND GAPS

It is convenient to write the free Euclidean inverse quark propagator G_0^{-1} at nonzero μ as a matrix acting on the column vector $\binom{\psi}{\bar\psi^T}$

$$G_0^{-1} = \begin{pmatrix} & C(q_\nu\gamma^\nu + i\mu\gamma^4)C \\ q_\nu\gamma^\nu - i\mu\gamma^4 & \end{pmatrix} \quad (1)$$

where C is the charge conjugation matrix. We write the full propagator of the interacting fermion system as

$$G = (G_0^{-1} + \Sigma)^{-1}, \quad (2)$$

with the proper self energy

$$\Sigma = \begin{pmatrix} \langle\psi\psi\rangle & -i\,\langle\overline{\psi}\psi\rangle \\ i\,\langle\overline{\psi}\psi\rangle & \langle\psi\psi\rangle \end{pmatrix}. \quad (3)$$

As a simple example, to demonstrate the physics in Figs. 1-4, consider two species of fermions forming the Lorentz scalar condensates

$$\langle\psi\psi\rangle = C\gamma_5 \begin{pmatrix} 0 & f \\ f & 0 \end{pmatrix}, \quad \langle\overline{\psi}\psi\rangle = \begin{pmatrix} 0 & 0 \\ 0 & M_s \end{pmatrix}. \quad (4)$$

The two fermions should be thought of as a massless u-quark with a certain color and an s-quark of a different color with constituent mass M_s. We will see in the next section that the ansatz (4) suffices to describe those features of the CFL phase of interest to us. The condensate f can be obtained by solving a self-consistent Schwinger-Dyson equation for Σ. Assuming a four-fermion interaction, this is given schematically by

where the loop denotes a momentum integration over the full propagator (2) and where the external legs have been amputated.

In order to determine the gap associated with a given condensate f, we need the quasiparticle dispersion relations [10]. These are determined by the poles of the full propagator (2). After some algebra one finds that the dispersion relations $\omega(\mathbf{q})$ are given by solutions of $D_+ = 0$ or $D_- = 0$, upon noting that $\omega = iq_0$, with

$$D_\pm = \left[q_0^2 + (\mu - |\mathbf{q}|)^2 + f^2)\right]\left[q_0^2 + (\mu + |\mathbf{q}|)^2 + f^2)\right]$$
$$-M_s^2\left[iq_0 \pm (\mu - |\mathbf{q}|)\right]\left[iq_0 \pm (\mu + |\mathbf{q}|)\right]. \quad (5)$$

It is instructive to consider the massless case, $M_s \to 0$. The dispersion relation is then $\omega(\mathbf{q}) = \pm\sqrt{(\mu \pm |\mathbf{q}|)^2 + f^2}$. The first $\pm$ distinguishes filled negative energy states from positive energy states describing excitations. The second $\pm$ distinguishes the particle/hole branch from the antiparticle branch. The particle/hole

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branch has a minimum of $\omega(\mu) = f$. There is a gap $2\Delta = 2f$ at the Fermi surface. We thus recover the familiar result that the gap and the condensate are equal if we pair quarks whose masses are degenerate. For $M_s \neq 0$, we obtain dispersion relations as illustrated in Figs. 1-4. We thus observe that when quarks with different masses pair, the condensate f yields quasiparticles with two different dispersion relations. The gaps for the two branches differ from each other and from f. For small enough f, gapless superconductivity results.

In the presence of any diquark condensate, the quasiparticles are linear combinations of particles and holes. In the presence of a $\langle us \rangle$ condensate, they are also linear combinations of u and s. We illustrate this by diagonalizing the propagator matrix (2), computing its eigenvectors, and thus determining the probability that the quasiparticles in Fig. 3 are u (particles or holes). The results, plotted in Fig. 5, demonstrate that the upper (lower) branch is mostly u (s) holes at low $|\mathbf{q}|$ and mostly s (u) particles at high $|\mathbf{q}|$, as must be the case given the sequence of Figs. 1-4. This allows us to explain the transition from Fig. 2 to Fig. 3 more clearly. In Fig. 2, because $\langle uu \rangle = \langle ss \rangle = 0$ there is no gap at the Fermi surface. In Fig. 3, the positive and negative energy branches which are close to touching at the Fermi surface both describe linear combinations of u and s. This means that the $\langle us \rangle$ condensate keeps them apart, and a true gap opens up.
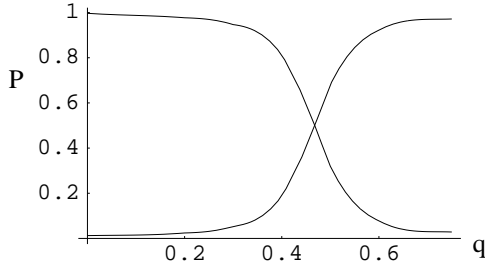
At sufficiently high density, QCD is in the CFL phase which, for $M_s \neq 0$, is characterized by an unbroken $SU(2)_{\text{color}+L+R}$ symmetry describing simultaneous $SU(2)$ rotations in color and vector flavor. In this phase, all nine quarks (3 colors times 3 flavors) form condensates. Thus, the ansatz (4) should be replaced by the 9×9 block-diagonal matrix of Ref. [7]. Four of the nine quarks form two doublets under $SU(2)_{\text{color}+L+R}$ and pairing between elements of these doublets results in $\langle us \rangle$ and $\langle ds \rangle$ condensates described by two 2×2 blocks, each with the form (4). Diagonal $\langle uu \rangle$ and $\langle ss \rangle$ entries in (4) do not arise because they break the $SU(2)_{\text{color}+L+R}$ symmetry; their presence would preclude gapless superconductivity. Three of the remaining five quarks, linear combinations of u and d only, pair among themselves and the resulting dispersion relations have gaps equal to the associated $\langle ud \rangle$ condensate. In the CFL phase, the last 2×2 block does involve $\langle us \rangle$ pairing but its entries are such that no dispersion relation can become gapless [7]. The only blocks from the full 9×9 ansatz of Ref. [7] which can yield gapless superconductivity are therefore the two copies of the ansatz (4).

In Ref. [7], we have solved the coupled mean-field Schwinger-Dyson equations for the CFL condensates including f. The only condensate of interest here is f, and we plot it in Fig. 6.
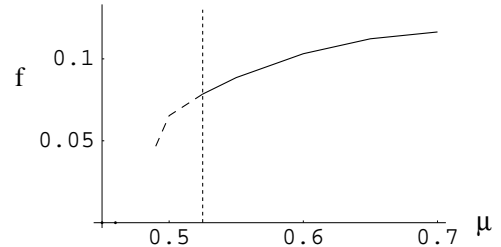

FIG. 5. Probability that a quasiparticle is u as opposed to s, for the lower two dispersion relations in Fig. 3.

III. METASTABLE GAPLESS SUPERCONDUCTIVITY

We follow Ref. [7] and describe superconducting strange quark matter in a model in which quarks interact via a four-fermion interaction abstracted from single-gluon exchange, $\mathcal{L}_{\text{int}} = G \int d^4 x \left(\overline{\psi} \lambda^a \gamma^\mu \psi \right) \left(\overline{\psi} \lambda^a \gamma_\mu \psi \right)$, with G chosen to give reasonable vacuum physics [7]. At asymptotically high densities, such phenomenological approaches have been superseded by calculations done using QCD itself [11]. As conjectured in [7], recent work [12,13] demonstrates that at accessible densities the two approaches are in reasonable agreement in their predictions for the magnitude of the condensates.

FIG. 6. f in the CFL phase as a function of μ for $M_s = 350$ MeV. For $\mu < 525$ MeV the stable superconducting phase has $f = 0$, and the CFL phase exists only as a metastable phase as indicated by the dashed line.

We can now explain the choice of parameters in Figs. 1-4. Fig. 4 depicts the dispersion relations at a generic point in the CFL phase, with $\langle us \rangle = f = 100$ MeV taken from Fig. 5 at $\mu = 600$ MeV. Fig. 3 gives the dispersion relations *at* the unlocking transition. The gap (15 MeV) is rather small compared to the condensate (85 MeV), but is still nonzero. At the unlocking transition, $\langle us \rangle = M_s^2/2.5\mu$ [7]. We find that gapless superconductivity sets in for $\langle us \rangle \leq M_s^2/3.8\mu$, in the metastable phase. Fig. 2 shows the dispersion relations for the metastable CFL phase below the unlocking transition at $\mu = 490$ MeV, $\langle us \rangle = 45$ MeV. In the model, as the simple arguments given in the introduction suggest, gapless superconductivity does not occur as the thermodynamic ground state. It does occur as a metastable phase.

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IV. IMPLICATIONS AND ANALOGUES

It is striking that the CFL phase breaks chiral symmetry [5] and has the same symmetries as sufficiently dense hypernuclear matter [6], so there need be no phase transition between them. It has been hypothesized [6] that there is "quark-hadron continuity": as μ is increased the baryonic condensates change continuously into quark condensates, and the gaps at the hyperon Fermi surfaces become gaps at the quark Fermi surfaces. This raises the possibility of obtaining information about the hadronic phase from calculations in the weakly-coupled quark phase [6,7]. Our results indicate, however, that it is in practice difficult to exploit quark-hadron continuity in this way, because the relative sizes of the *gaps* in the various channels may be quite different in the hadronic and quark regimes, even if the relative sizes of the *condensates* are similar. The reason is that, as shown above, in channels where there is pairing of fermions of different mass, the gaps depend sensitively on the fermion masses as well as on the condensates. However, the fermion masses change dramatically in magnitude and in pattern as μ is increased from the hypernuclear phase into the quark phase. In the quark phase, there are six light quarks and three heavier strange quarks, which is quite unlike the pattern of masses for the baryons. We conclude that it will be hard to infer physics of hadronic matter, such as the ratio of the gap in one channel to that in another, from calculations performed in the quark matter regime.

Pairing between fermions with different dispersion relations occurs in other contexts. Analogues include pairing between neutrons and protons in nuclear matter which is not isospin symmetric [14], and pairing between spin-up and spin-down electrons in an ordinary superconductor placed in a magnetic field H which introduces a Zeeman splitting $\mu_B H$ [15,16]. The latter case yields a particularly apt analogue. In some superconductors in which the spin-orbit coupling is small, gapless superconductivity would set in for $\mu_B H > \Delta$, where Δ is the gap at $H = 0$ [15,16]. However, what happens instead is that a first order phase transition to a nonsuperconducting phase occurs at $\mu_B H = \Delta/\sqrt{2}$ [15,16]. This is precisely analogous to what we have found: unlocking precludes gapless superconductivity, except as a metastable phase. It is worth noting that if one introduces paramagnetic impurities, instead of a uniform magnetic field, there is a range of impurity concentrations for which the superconductor *is* gapless [17,16]. This example suggests that although in our model gapless superconductivity only occurs below the unlocking transition, where the CFL phase is metastable, in QCD itself it may occur for a range of chemical potentials above the unlocking transition, where the CFL phase is the stable thermodynamic ground state.

The thermodynamic properties of quark matter in the CFL phase need not be characterized by a gap. Gapless superconductivity is most likely to occur at the lowest densities at which the CFL phase is present. This makes it of potential interest in neutron star phenomenology. Further work is required to elucidate the importance of this observation, because the quasiparticles which we have analyzed are not the only excitations in the CFL phase. The Nambu-Goldstone bosons arising from the spontaneous breaking of global symmetries (chiral symmetry and $U(1)_B$) and the massive gluons arising from the breaking of gauge symmetries will also contribute to the specific heat and to transport properties. It remains to be seen how significant the presence of quasiparticles with a gapless dispersion relation would be in the context of neutron star phenomenology.

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